

## MAT601: FIELDS AND GALOIS THEORY

**Unit 1:** Eisenstein's irreducibility criterion, Characteristic of a field, Prime subfields, Field extensions, Finite extensions, Simple extensions, Algebraic and transcendental extensions. Factorization of polynomials in extension fields.

**Unit 2:** Splitting fields and their uniqueness. Separable field extensions, Perfect fields, Separability over fields of prime characteristic, Transitivity of separability.

**Unit 3:** Automorphisms of fields, Dedekind's theorem, Fixed fields, Normal extensions, Splitting fields and normality, normal closures, Galois extensions, Fundamental theorem of Galois theory, Computation of Galois groups of polynomials.

**Unit 4:** Primitive element theorem, Finite fields, Existence and uniqueness, Subfields of finite fields, Characterization of cyclic Galois groups of finite extensions of finite fields, fundamental theorem of algebra.

~~**Unit 5:** Cyclotomic extensions and polynomials, cyclic extensions, Solvability by radicals, Galois' characterization of such solvability, Generic polynomials, Abel-Ruffini theorem, geometrical constructions.~~

### Books Recommended:

1. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley & Sons, N.Y., 2003.
2. N. S. Gopalakrishnan, University Algebra, Wiley Eastern, New Delhi, 1986.
3. T. W. Hungerford, Algebra, Springer (India), Pvt. Ltd., 2004.

प्रो. अम

M. S. Singh  
26.9.16

M. K. L.  
26.9.16

M. S. Singh  
26.9.16

R. P. S. N.

...

P. K. Singh  
26.9.16

A. Chatur

M. K. L.  
अध्यक्ष  
20.10.2020

Head

अभिलेख विभाग

Department of Mathematics  
इलाहाबाद विश्वविद्यालय  
University of Allahabad  
Allahabad-201002

M. K. L.

MAT603: FUNCTIONAL ANALYSIS

UNIT ONE: Normed linear spaces, examples and its topological properties, Banach spaces, continuous linear transformations, spaces of continuous linear transformations from a linear space to a Banach space, continuous linear functionals.

UNIT TWO: Hahn-Banach theorem, Open mapping theorem, Closed graph theorem, Banach-Steinhaus theorem (or the Uniform boundedness principle).

UNIT THREE: Conjugate spaces, natural embedding of  $N$  in  $N^{**}$ , weak and weak\*-topology on a conjugate space, conjugate of an operator, simple applications to reflexive separable spaces.

UNIT FOUR: Hilbert Spaces, Schwarz's inequality, orthogonal complement, Bessel's inequality, orthonormal sets, continuous linear functionals on Hilbert spaces, Riesz representation theorem, reflexivity of Hilbert Spaces, adjoint of an operator on a Hilbert space.

~~UNIT FIVE: Self-adjoint and normal operators, unitary operators on a Hilbert space, projections on Hilbert spaces, determinant and the spectrum of an operator, spectral theorem.~~

Books Recommended:

1. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill, 1963.
2. S. Ponnusamy, Foundations of Functional Analysis, Narosa Publishing House, New Delhi, 2002.

Prash

M. S. Jadhav  
26/9/16

M. K. L.  
26.9.16

M. S. J.  
26/9/16

R. S. M.

S. S. J.  
26.9.16

M. K. L.  
15.10.2020

अध्यक्ष  
Head

गणित विभाग  
Department of Mathematics  
इलाहाबाद विश्वविद्यालय  
University of Allahabad  
Allahabad-201002

A. H. M.  
26/9/16

A. Chatterjee

A. H. M.

# MAT605: THEORY OF ORDINARY DIFFERENTIAL EQUATIONS

## UNIT ONE:

Initial and Boundary Value Problems, Picard's Iterations, Lipschitz conditions, Sufficient conditions for being Lipschitzian in terms of partial derivatives, Examples of Lipschitzian and Non-Lipschitzian functions, Picard's Theorem for local existence and uniqueness of solutions of an initial value problem of first order which is solved for the derivative, examples of problems without solutions and of equations where Picard's iterations do not converge, Differential equations of first order not solved for the derivative, Uniqueness of solutions with a given slope, Singular solutions,  $p$ - and  $c$ -discriminant equations of a differential equation and its family of solutions respectively, Envelopes of one parameter family of curves, singular solutions as envelopes of families of solution curves, Sufficient conditions for existence and nonexistence of singular solutions, examples.

## UNIT TWO:

Systems of I order equations arising out of equations of higher order, Norm of Euclidean spaces convenient for analysis of systems of equations, Lipschitz condition for functions from  $R^{n+1}$  to  $R^n$ , Local existence and uniqueness theorems for systems of I order equations, Gronwall's inequality, Global existence and uniqueness theorems for existence of unique solutions over whole of the given interval and over whole of  $R$ , Existence theory for equations of higher order, Conditions for transformability of a system of I order equations into an equation of higher order.

## UNIT THREE:

Linear independence and Wronskians, General solutions covering all solutions for homogeneous and non-homogeneous linear systems, Abel's formula, Method of variation of parameters for particular solutions, Linear systems with constant coefficients, Matrix methods, Different cases involving diagonalizable and non-diagonalizable coefficient matrices, Real solutions of systems with complex eigenvalues.

## UNIT FOUR:

Convergence of real power series, Radius and interval of convergence, Ordinary and singular points, Power series solutions, Frobenius' generalized power series method, Indicial equation, different cases involving roots of the indicial equation, Regular and logarithmic solutions near regular singular points.

## UNIT FIVE:

Legendre's equation, Solution by power series method, polynomial solution, Legendre polynomial, Rodrigues' formula, Generating function, Recurrence relations, Orthogonality relations, Fourier-Legendre expansion, Bessel's equation, Bessel functions of I and II kind, Recurrence relations, Bessel functions of half-integral orders, Sturm comparison theorem, Zeros of Bessel functions, Orthogonality relations, Generating function, Fourier-Bessel expansions.

Mul 15.10.2020  
अध्यक्ष  
Head  
गणित विभाग  
Department of Mathematics  
इलाहाबाद विश्वविद्यालय  
University of Allahabad  
इलाहाबाद  
Department of Mathematics  
University of Allahabad  
इलाहाबाद विश्वविद्यालय  
Head  
गणित विभाग  
अध्यक्ष

Mul 26.9.16  
16  
PHD 26/9/16  
MISS 26/9/16  
Sutras  
26.9.16  
RPSL  
CA. Thakur

Books Recommended:

1. B. Rai, D. P. Choudhury and H. I. Freedman, A Course in Ordinary Differential Equations, Narosa Publishing House, New Delhi, 2002.
2. E. A. Coddington, An Introduction to Ordinary Differential Equations, Prentice Hall of India, New Delhi, 1968.

MK  
26/9/16

Mk  
26.9.16

M. Singh  
26/9/16

Satyam  
26.9.16

RBSN

AK

1

R. Pandey  
26/9/16

A. Chatterjee

M. K. S. 10.2020  
अध्यक्ष

Head

गणित विभाग

Department of Mathematics

इलाहाबाद विश्वविद्यालय

University of Allahabad

इलाहाबाद, Allahabad.

P. Anand

MAT 607: FLUID MECHANICS

UNIT ONE:

Review of basic concepts, Real and ideal fluids, Newton's law of viscosity, Convective transport of scalar and vector quantities, Differentiation following the motion and acceleration, The equation of Continuity, Velocity Potential, Body forces, surface forces, Stress vector at a point, Nature of stresses, Stress on an Arbitrary Plane: Cauchy's Stress formula, State of stress at a point, Stress tensor, Isotropic Law of Pressure, Principal stresses and Principal Directions, Stress invariants, General displacement of a fluid element.

UNIT TWO:

Nature of strains, Rates of strain components, Relation between stress and rates of strain, Transformation of Stress-Components, Transformation of Rates of Strain, Navier-Stokes equation, Euler's Equation, Energy Dissipation due to Viscosity, Diffusion of Vorticity,

UNIT THREE

Stream tube and Vortex tube, Helmholtz's vorticity theorem, Kelvin's Circulation theorem, Energy Flux, Mean Potential over a spherical surface in a simply connected region, Kinetic Energy in Irrotational Flow, Kelvin's Minimum kinetic energy Theorem, Uniqueness of the Irrotational motion

UNIT FOUR:

Two dimensional irrotational motion, The stream function, The Complex potential for two dimensional irrotational motion, Concept of line-sources, sinks, doublets and vortices, Superposition of solutions, The concept of Images, The Vortex pair, Vortex rows: Single infinite row of Line Vortices, the Karman vortex street, Milne-Thomson Circle Theorem, Blasius Theorem, Complex potential for a uniform flow past a circular cylinder, Streaming and circulation about a fixed circular cylinder, Conformal transformation: Uniform line distributions (source, vortex and doublet) under conformal transformation.

UNIT FIVE:

Three dimensional irrotational flow, Concept of Sources, Sinks and Doublets, Axisymmetric flows, Stokes stream function, Statements of Weiss's and Butler's sphere theorems and their applications, Liquid streaming past a stationary sphere, Uniform motion of a sphere in a liquid at rest at infinity, Gravity waves - Surface waves on the infinite free surface of liquids, Waves at the interface between finitely and infinitely deep liquids.

Books Recommended:

1. L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Butterworth-Heinemann, 2<sup>nd</sup> Edition, 1987.
2. N. Curle and H. J. Davies, Modern Fluid Dynamics, Vol. I, D. van Nostrand Comp. Ltd., London, 1968.
3. S. W. Yuan, Foundations of Fluid Mechanics, Prentice-Hall, Englewood Cliffs, NJ, 1967.
4. A. S. Ramsey, A Treatise on Hydrodynamics, Part I, G. Bell and Sons Ltd. 1960.
5. F. Chorlton, Text Book of Fluid Dynamics, CBS Publishers, Delhi, 1985

M. S. C. 16.2020  
अध्यक्ष  
Head  
विभागाध्यक्ष  
Department of Mathematics  
आलहाबाद विश्वविद्यालय  
University of Allahabad  
Allahabad-211007  
A. Ahmad

haha

दिनांक 26/9/16

M. S. C. 26.9.16

M. S. C. 26/9/16

S. S. C. 26.9.16

26/9/16

RRCA

## MAT 651: RIEMANNIAN GEOMETRY

Unit 1: Riemannian metrics, Riemannian manifolds, examples, Levi-Civita connection, fundamental theorem of Riemannian geometry, Curvature tensors- Riemannian curvature tensor, sectional curvature, Schur's Theorem, Ricci curvature, scalar curvature, Einstein manifolds.

Unit 2 : Gradient vector fields, divergence of a vector field, Covariant derivative along a curve, parallel transport, length of a curve. Distance function, geodesics, Exponential map,

Unit 3: Jacobi fields, Gauss Lemma, complete Riemannian manifolds, Hopf -Rinow Theorem, The theorem of Hadamard, Riemannian immersions, second fundamental form, Gauss equation, Model spaces of constant curvature.

Unit 4: Lie derivative, Lie derivatives of scalars, vectors, tensors and linear connections, commutation formula for Lie differential operator and covariant differential operator.

~~Unit 5: Motion, Affine motion, projective motion in a Riemannian space, curvature collineation, conformal and homothetic transformations.~~

### Books Recommended:

1. M. P. do Carmo; Riemannian Geometry, Berkhauser, 1992.
2. P. Peterson; Riemannian Geometry, Springer, 2006.
3. J. Jost; Riemannian Geometry and Geometric Analysis, Springer, (6<sup>th</sup> edition), 2011.
4. J. M. Lee; Riemannian Manifolds: An Introduction to Curvature, Springer, 1997.
5. S. Gallot, D. Hullin. J. Lafontaine; Riemannian Geometry, Springer (3<sup>rd</sup> edition), 2004.
6. K. Yano; The Theory of Lie derivatives and its Applications, North Holland Publishing Company, Amsterdam, 1957.

Handwritten signatures and dates:   
MSM 26/9/16   
MEL 26-9-16   
MUSING 26/9/16   
SUTJEN 26.9.16   
RDSM   
A. Chatterjee   
26/9/16

MAL 15-10-2020  
अध्यक्ष  
Head  
अभिलेखित विभाग  
Department of Mathematics  
इलाहाबाद विश्वविद्यालय  
University of Allahabad  
इलाहाबाद/Allahabad